76th MORSS 10-11-12 JUNE 2008

MORS Military Operations Research Society

GOVERNMENT DISCLOSURE FORM 712B

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Principal Author: 2Lt. Anthony Alvin Illig

Other Author(s): Maj. August Roesener, Maj. Shane Knighton, Maj. Shane Hall

Principal Author's Organization: Air Force Institute of Technology

Complete mailing address: 346 Evergreen Ave. Niceville, FL 32578

X [signature]

Principal Author's Signature: Date: 29 Apr 08

Phone: (850) 883-5010

FAX: (850) 882-6277

Email: anthony.illig@afotec.af.mil

Title of Presentation: Hurry up and Wait ... Optimizing the Initial Skills Training Scheduling Process

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Releasing Official's title: Public Affairs Asst

X [signature] Sandra J. Simison

Releasing Official's Signature:

Printed name: Sandra J. Simison

Organization: AFIT/PA

Date: 30 Apr 08

Complete mailing address: 2950 Hobson Way WPAFB OH 45433-7765

Phone:

FAX:

Report Documentation Page

Report Documentation Page	Form Approved OMB No. 0704-0188

1. REPORT DATE	2. REPORT TYPE	3. DATES COVERED
01 JUN 2008	**N/A**	-

4. TITLE AND SUBTITLE	
A Network Flow Approach To The Initial Skills Training Scheduling Problem	5a. CONTRACT NUMBER
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **AFIT**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S)
	11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release, distribution unlimited

13. SUPPLEMENTARY NOTES

See also ADM202527. Military Operations Research Society Symposium (76th) Held in New London, Connecticut on June 10-12, 2008, The original document contains color images.

14. ABSTRACT

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			
unclassified	**unclassified**	**unclassified**	**UU**	**33**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

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A NETWORK FLOW APPROACH TO THE INITIAL SKILLS TRAINING SCHEDULING PROBLEM

Lt. Tony Illig

Maj. August Roesener

Maj. Shane Knighton

Maj. Shane Hall

Overview

- Problem Description
- Motivation
- Network Flow Approach
- Why Network Flow Works
- Results
- Future Research

Problem Description

- Initial Skills Training Scheduling Problem (ISTSP):
  - After commissioning, new Air Force officers typically require training to be certified to perform their duties.
  - Commissioning sources: USAFA, ROTC, OTS
  - All officers must attend ASBC. (Mandated by CSAF)
  - Following ASBC, many officers continue training to be certified
    - Pilots
    - Combat Systems Officers (CSOs)
    - Air Battle Managers
    - Intel Officers
    - Space and Missile Officers
    - Air Field Operations Officers

Problem Description

- Initial Skills Training Scheduling Problem (ISTSP):
  - Course sequences are rigid and vary by AFSC
  - Class sizes and intervals vary
  - Course length is fixed
  - Class blends must be met
    - AETC desires that certain blends are met according to commissioning source and AFSC (in ASBC).

Motivation

- Average 2nd Lt Pay:
  - $124 / day   (2007 dollars)
- Number of Annually Commissioned 2nd Lts:
  - ~ 4000
- Estimated savings of 1 day:
  - $496,000
- Current average number of down days:
  - ~ 180
- Huge Potential for Savings!!!!

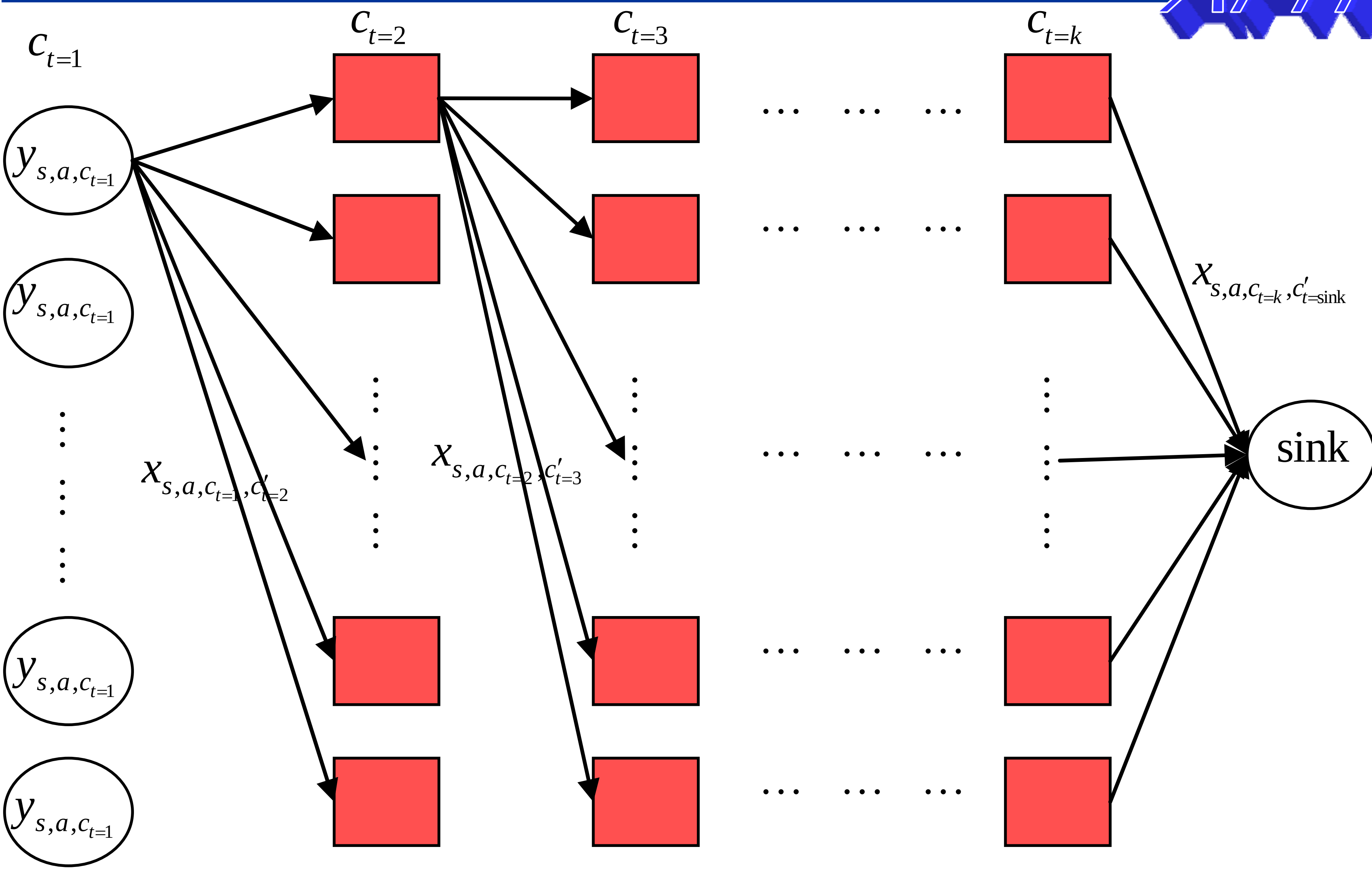
Network Flow Approach

- General personnel scheduling techniques are computational intensive
- ISTSP has structure which can be exploited
- Developed network flow approach (NFISTSP)
  - How it works
    - Commissioning sources are source nodes.
    - Create nodes to represent each class.
    - Connect the nodes with arcs if it is possible to go from the starting node’s class to the ending nodes class.
    - The cost on the arc is the number of days between the classes.
    - Use minimum cost flow to solve.

NFISTSP Approach

- What it accomplishes
  - Enforces blends:
    - Source of commissioning (SOC) blends enforced by creating 3 separate nodes at each class (one for each SOC) then capacitating arcs accordingly.
    - AFSC blends are done in a similar way at ASBC.
  - Does not allow skipping.

NFISTSP Graph



NFISTSP Graph

Sum all nodes for class UB

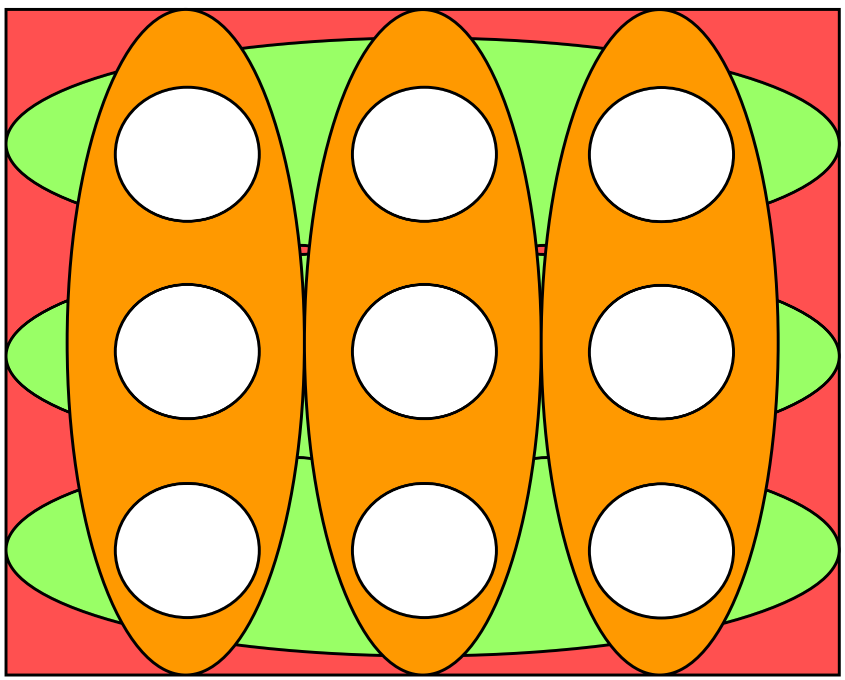
$$\sum_{\forall s\in \mathbf{S}} \sum_{a\in \mathbf{A}'} \sum_{\forall c\in \mathbf{C}_t} x_{s,a,c,c'} \le UB_{c'}$$

Sum all nodes for AFSC UB

$$\sum_{\forall s\in \mathbf{S}} \sum_{\forall c\in \mathbf{C}_t} x_{s,a,c,c'} \le UB_{a,c'}$$

Sum all nodes for SOC UB

$$\sum_{\forall a\in \mathbf{A}'} \sum_{\forall c\in \mathbf{C}_t} x_{s,a,c,c'} \le UB_{s,c'}$$



Why does it solve quickly?

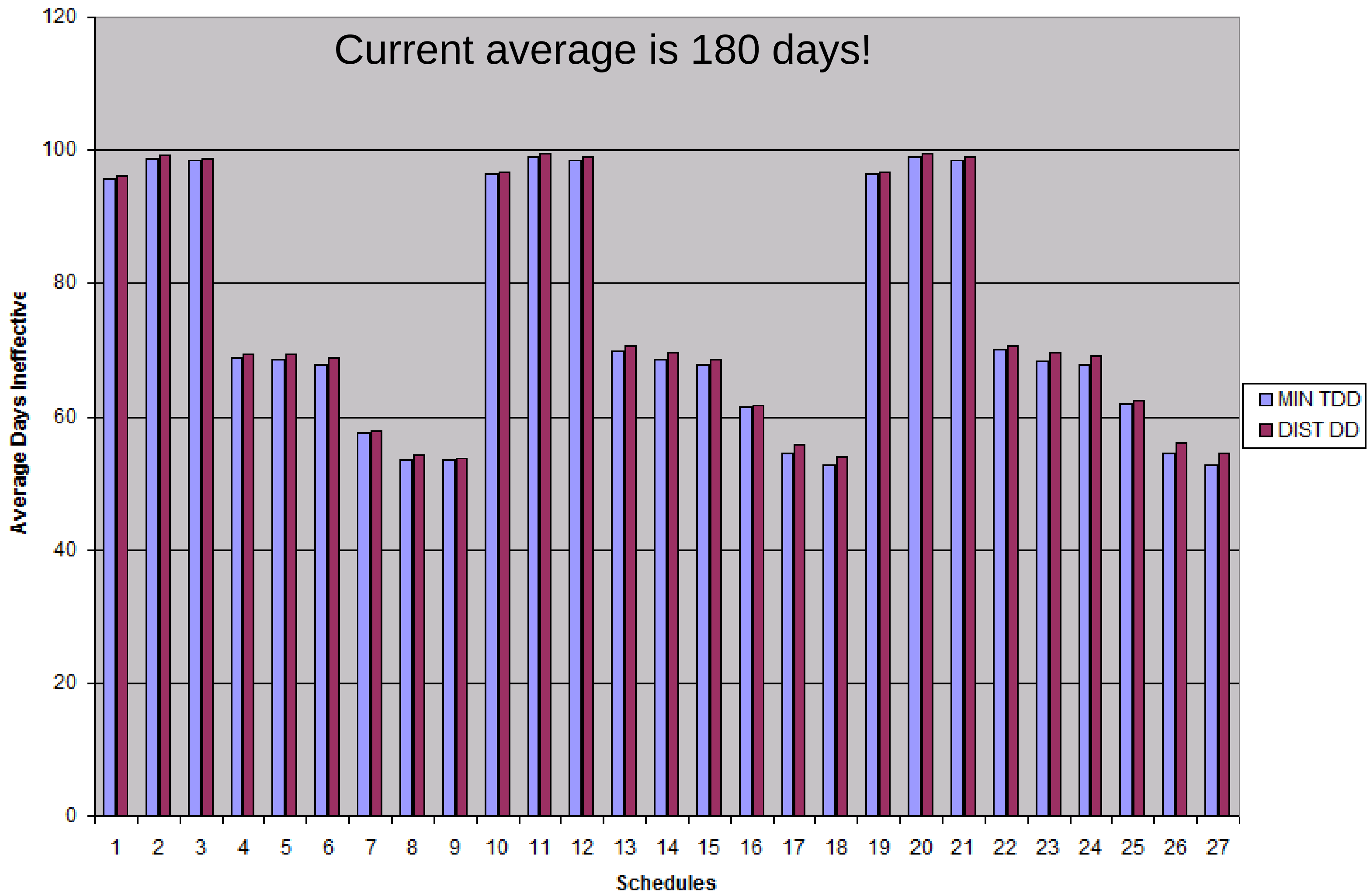
- Using this formulation allows large personnel scheduling problems to solve in seconds
  - Why?
- Total Unimodularity (TU)
  - Definition: ***A*** is totally unimodular iff every square sub matrix in A has a determinant of -1, 0 or 1.
  - Implications: Suppose ***A*** is the constraint matrix for a integer programming (IP) problem. If ***A*** is TU and the RHS is all integer <u>then the LP relaxation of the IP will yield integer results.</u>

Test Cases/Scenarios

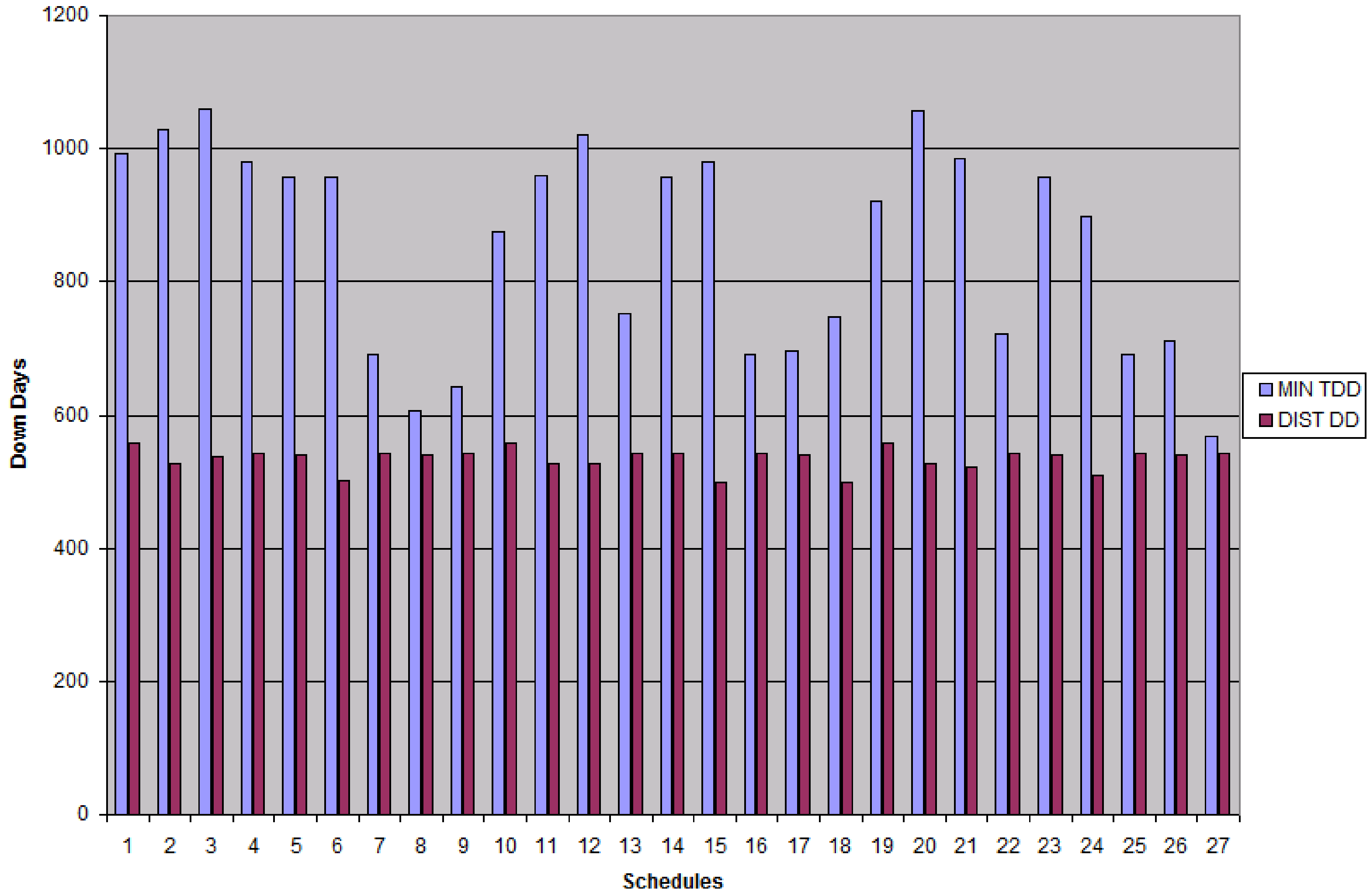
- Min USAFA Leave
  - 0, 30, 60 Days — 3 Levels
- Max ROTC Delay
  - 0, 180, 365 Days — 3 Levels
- Class Blend Levels
  - 50%, 75%, No Forced Blend — 3 Levels
- Arc Weighting Schemes (OF)
  - Minimize TDD, Distribute DD — 2 Levels

Total Scenarios: 54

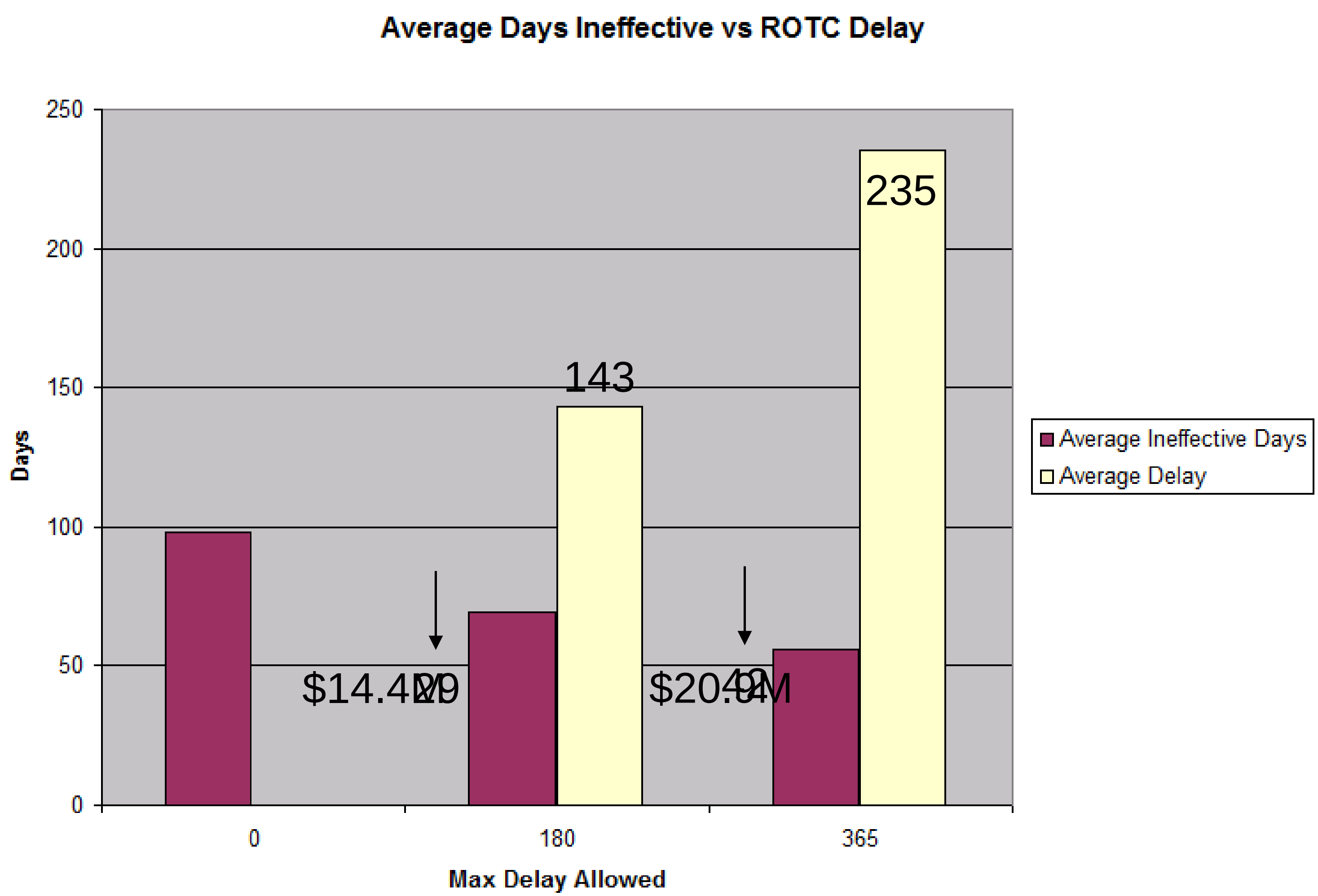
Minimize TDD vs Distribute DD



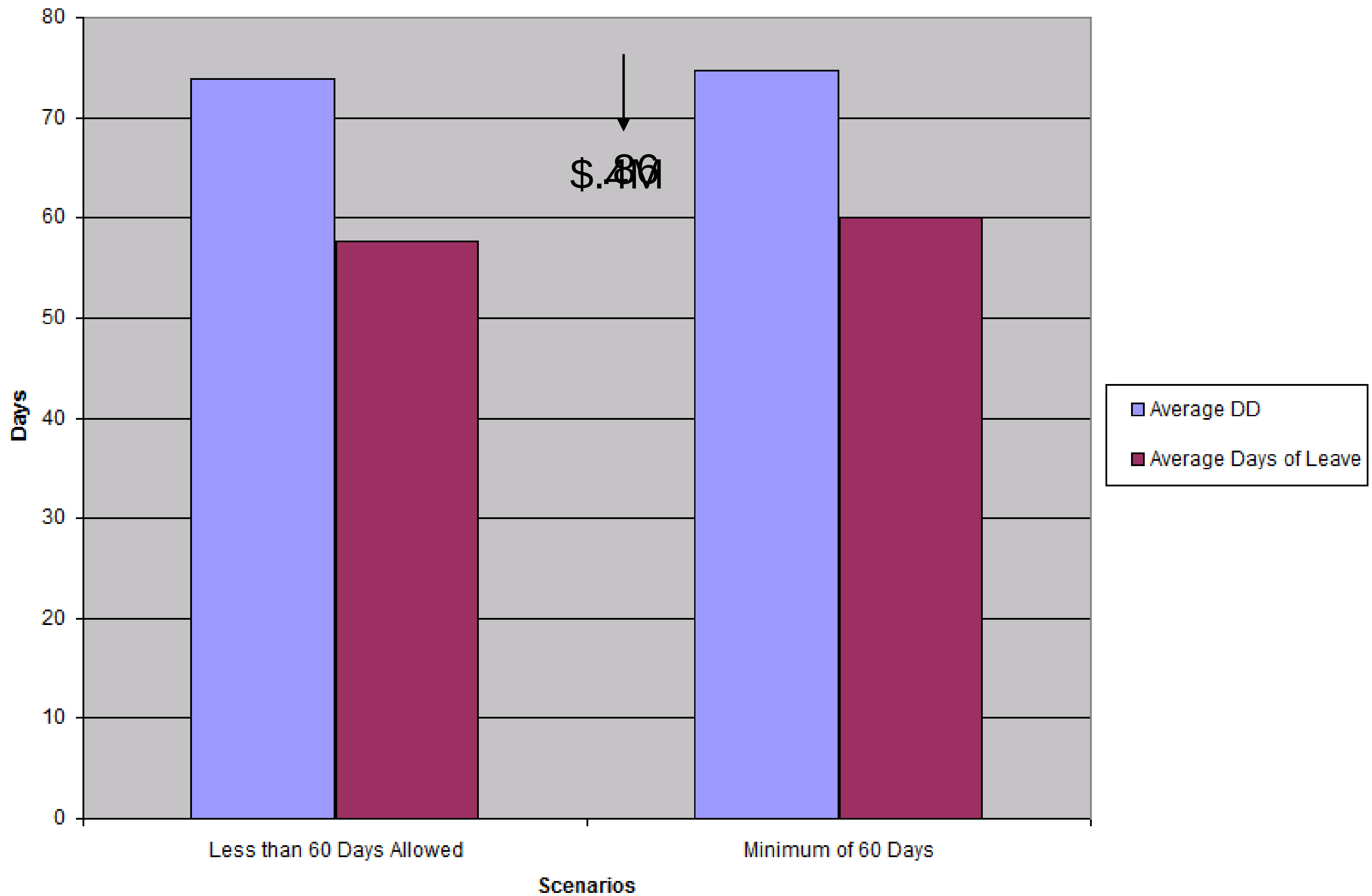
Minimize TDD vs Distribute DD



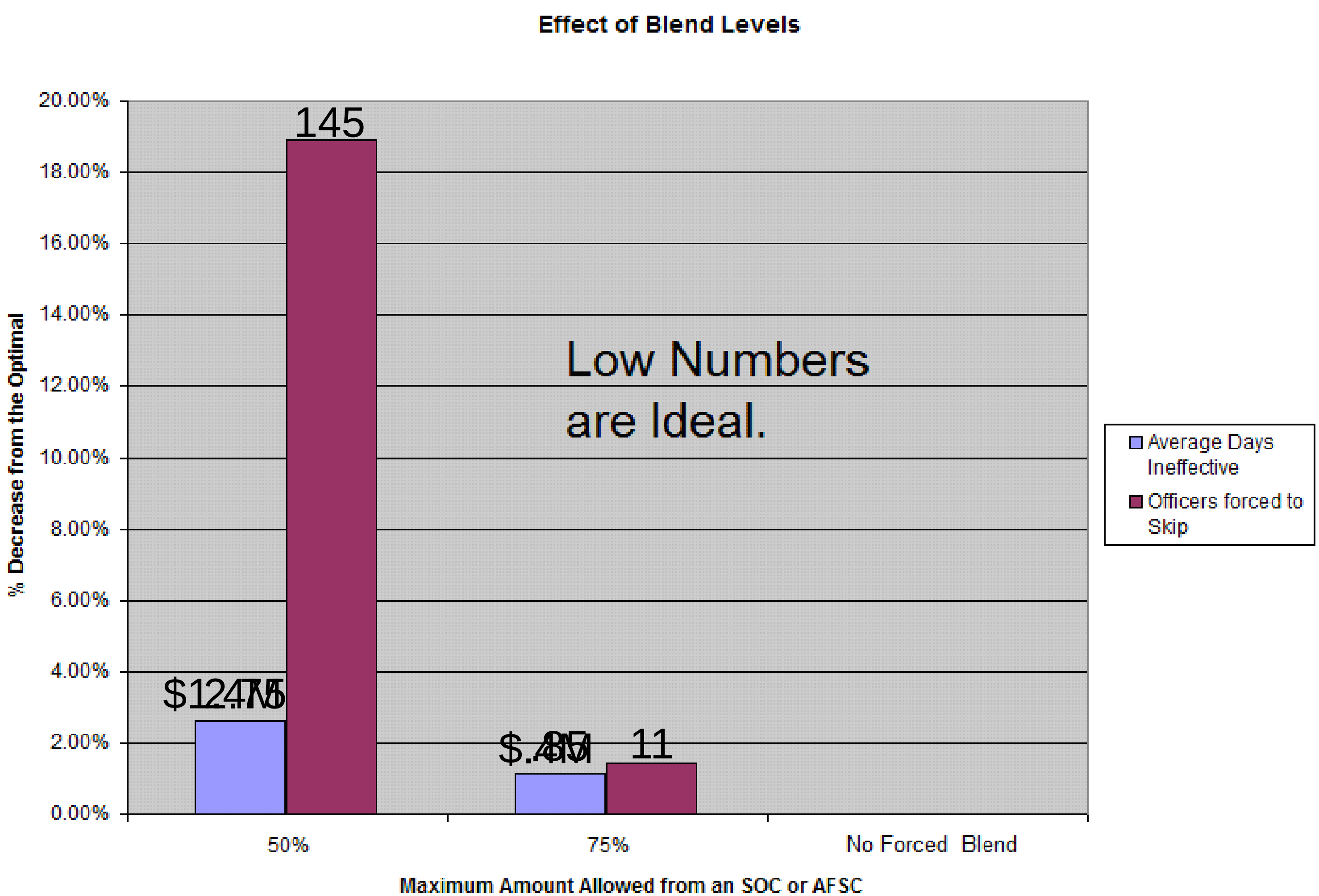
Impact of ROTC Delay



Impact of USAFA Leave



Impact of Blend Levels



Computational Conclusions

- The savings (dollars or ineffective days) from reducing the minimum USAFA Leave is not significant.
- ROTC delay extension results in a sizable effect.
- Restricting blends significantly impacts total number of down days; partial blends occur naturally.

Robustness of Formulation

- Theoretically, any timeline can be scheduled.

- This formulation handles heterogeneous workforces well.
  - In this example we had 2 heterogeneous levels (AFSC and SOC).

  - Can handle any number of types and levels efficiently.

Future Research

- Use of Decision Analysis
- Use new optimization package in SAS/OR
- Use model to schedule optimal class sizes and class start dates
- Use probabilities and stochastic analysis to include drop out rates

Questions?

Thanks for your time …

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NFISTSP Mathematical Formulation

- Sets

$$\mathbf{S} = \text{set of all commissioning sources} = \{s_1, s_2, \ldots, s_n\}$$

$$\mathbf{A} = \text{set of all AFSCs} = \{a_1, a_2, \ldots, a_m\}$$

$$\mathbf{T} = \text{set of all training courses} = \{t_1, t_2, \ldots, t_k\}$$

$$\mathbf{C}_t = \text{set of all classes of training course } t = \{c_{t_1}, c_{t_2}, \ldots, c_{t_l}\}$$

- Variables

$$x_{s,a,c_t,c_t'} = \text{number of officers from commissioning source } s \text{ in AFSC } a \text{ going from class } c_t \text{ to class } c_t'$$

NFISTSP Mathematical Formulation

- Parameters

$$d_{c_t,c_t'} = \text{number of days bettween class } c_t \text{ and class } c_t'$$

$$y_{s,a,c_t} = \text{number of officers from commissioning source } s$$
$$\text{in AFSC } a \text{ graduating from class } c_t$$

- Objective Function

$$\text{Minimize} \qquad Z = \sum_{\forall t \in T} \sum_{\forall c_t' \in \mathbf{C}_{t+1}} \sum_{\forall c_t \in \mathbf{C}_t} d_{c_t,c_t'} \sum_{\forall a \in \mathbf{A}} \sum_{\forall s \in \mathbf{S}} x_{s,a,c_t,c_t'}$$

NFISTSP Mathematical Formulation

- Subject to

$$y_{s,a,c_1} = \sum_{\forall c' \in \mathbf{C}_2} x_{s,a,c_1,c'_2} \qquad \forall s \in \mathbf{S}, a \in \mathbf{A}, c_1 \in \mathbf{C}_1$$

$$\sum_{\forall c_t \in \mathbf{C}_t} x_{s,a,c_t,c'_{t+1}} = \sum_{\forall c'_{t+2} \in \mathbf{C}_{t+2}} x_{s,a,c_{t+1},c'_{t+2}} \quad \forall s \in \mathbf{S}, a \in \mathbf{A}, c'_{t+1} = c_{t+1} \in \mathbf{C}_{t+1}, t = 1,2,\ldots,n-1$$

$$\sum_{\forall s \in \mathbf{S}} \sum_{\forall a \in \mathbf{A}} \sum_{\forall c_k \in \mathbf{C}_k} x_{s,a,c_k,c'} = \sum_{\forall s \in \mathbf{S}} \sum_{\forall a \in \mathbf{A}} \sum_{\forall c_t \in \mathbf{C}_1} y_{s,a,c_1} \qquad \mathrm{c}' = \text{"sink"}$$

$$\sum_{\forall s \in \mathbf{S}} \sum_{a \in \mathbf{A}'} \sum_{\forall c_t \in \mathbf{C}_t} x_{s,a,c_t,c'_{t+1}} \le UB_{c'_{t+1}} \qquad \forall c'_{t+1} \in \mathbf{C}_{t+1}, t = 1,2,\ldots,n-1$$

$$\sum_{\forall s \in \mathbf{S}} \sum_{\forall c_t \in \mathbf{C}_t} x_{s,a,c_t,c'} \le UB_{a,c'_2} \qquad \forall a \in \mathbf{A}, c'_2 \in \mathbf{C}_2$$

$$\sum_{\forall a \in \mathbf{A}'} \sum_{\forall c_t \in \mathbf{C}_t} x_{s,a,c_t,c'_{t+1}} \le UB_{s,c'_{t+1}} \qquad \forall s \in \mathbf{S}, c' \in \mathbf{C}_{t+1}, t = 1,2,\ldots,n-1$$

Properties of Total Unimodularity

- Properties of TU that will be useful:
  - If A is TU then A^T is TU
  - If A is TU then (A, I) is TU
  - Row and Column swaps in A do not affect TU.
  - The Node-Arc incidence matrix of a Network Flow problem is always TU
  - If A is TU and B is obtained by removing a row or column of A, then B is TU
- The first four are from Bazaraa, Jarvis and Sherali

Proving Properties of Total Unimodularity

- If A is TU and B is obtained by removing a row or column of A, then B is TU

$$\begin{bmatrix} \cancel{a_{11}} & \cancel{a_{12}} & \cancel{a_{13}} & \cancel{a_{14}} & \cancel{\cdots} & \cancel{a_{1j}} \\ a_{21} & a_{22} & a_{23} & a_{24} & \cdots & a_{2j} \\ a_{31} & a_{32} & a_{33} & a_{34} & \cdots & a_{3j} \\ a_{41} & a_{42} & a_{43} & a_{44} & \cdots & a_{4j} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & a_{i4} & \cdots & a_{ij} \end{bmatrix}$$

Total Unimodularity

- A look at the constraint matrix for a network (the node incidence matrix)

$$
\begin{array}{cc}
 & \text{Arcs} \\
\text{Nodes} &
\begin{bmatrix}
a_{11} & a_{12} & a_{13} & a_{14} & \cdots & a_{1j} \\
1 & -1 & 0 & 0 & \cdots & 0 \\
-1 & 1 & 1 & 0 & \cdots & 0 \\
0 & 0 & -1 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
a_{i1} & a_{i2} & a_{i3} & a_{i4} & \cdots & a_{ij}
\end{bmatrix}
\end{array}
$$

Total Unimodularity

- Another sufficient condition for TU is:
  - For every J $\subseteq$ N={1,…,n}, there exists a partition J_1, J_2 of J such that

$$\left| \sum_{j \in J_1} a_{ij} - \sum_{j \in J_2} a_{ij} \right| \leq 1 \qquad \forall i = 1, \ldots, m$$

Total Unimodularity

- The side constraints for this problem are of this form:

$$C = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & \cdots & a_{1j} \\ a_{21} & a_{22} & a_{23} & a_{24} & \cdots & a_{2j} \\ a_{31} & a_{32} & a_{33} & a_{34} & \cdots & a_{3j} \\ a_{41} & a_{42} & a_{43} & a_{44} & \cdots & a_{4j} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & a_{i4} & \cdots & a_{ij} \end{bmatrix}$$

$$C \in \{0,1\}^{mxn}$$

Total Unimodularity

- The side constraints for this problem are of this form:

$$C' = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & \cdots & a_{1j} \\ a_{21} & a_{22} & a_{23} & a_{24} & \cdots & a_{2j} \\ a_{31} & a_{32} & a_{33} & a_{34} & \cdots & a_{3j} \\ a_{41} & a_{42} & a_{43} & a_{44} & \cdots & a_{4j} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & a_{i3} & a_{i4} & \cdots & a_{ij} \end{bmatrix}$$

$$C' \in \{1,0\}^{mxn}$$

Total Unimodularity

- Consider the Matrix:

$$\begin{array}{r} \text{Network} \longrightarrow \\[2em] \text{0,1 Matrix} \longrightarrow \\[2em] \text{Compliment} \longrightarrow \end{array} \begin{bmatrix} A \\[2em] C \\[2em] C' \end{bmatrix} \begin{array}{l} \longrightarrow \text{Set } J_1 \\[2em] \longrightarrow \begin{array}{l} \text{Odd rows to Set } J_1 \\ \text{Even rows to Set } J_2 \end{array} \\[2em] \longrightarrow \begin{array}{l} \text{Odd rows to Set } J_1 \\ \text{Even rows to Set } J_2 \end{array} \end{array}$$

Total Unimodularity

- Another sufficient condition for TU is:
  - For every J $\subseteq$ N={1,…,*n*}, there exists a partition J_1, J_2 of J such that

$$\left| \underbrace{\sum_{j \in J_1}}_{n/2 \text{ or } (n+1)/2} a_{ij} - \underbrace{\sum_{j \in J_2}}_{n/2 \text{ or } (n-1)/2} a_{ij} \right| \leq 1 \qquad \forall i = 1, \ldots, m$$

Total Unimodularity

- Therefore, this matrix is TU

$$\begin{bmatrix} A \\ C \\ \cancel{C'} \end{bmatrix}$$